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whole of which were received from Madras, and of eighteen of which from Greenwich. Lieut. Tennant finds a small outstanding discordance in his two results, when the two authorities for the stars are separately used, which for the present he leaves undecided, giving the preference temporarily to the results from the Madras stars, for reasons stated in the MSS.

The latitude of the Karachi geodesical station thus found (from 807 observations) was—

	° ′ ″
By Madras positions of stars	24 49 49·3
Greenwich	50·0

The azimuth of the referring mark was found to be,

	° ′ ″
By E. and W. elongations of *Polaris*	179 59 57·4
Same as brought up from Kalianpur	57·7

The intermediate results of the calculations are given in tables at much length.

An Examination of the Figure of the Indian Meridian as deduced by the Venerable Archdeacon Pratt from the two Northern Indian Arcs; with a Proposition for testing that form by Astronomical Observation. By Lieut. J. F. Tennant, Bengal Engineers, F.R.A.S.

1. On my arrival in May last at the Surveyor-General's Office, I met with Mr. Pratt's paper on Himalayan Attraction in the *Philosophical Transactions*, and it occurred to me that the arc of longitude from Kalianpur to Karachi completed in 1853 would furnish a means of verifying its results.

2. When, however, I applied the data given by Mr. Pratt to it and the southern meridianal arc, I became convinced that some error had been committed in the solution of the equations, and I therefore examined this with the following result.

The mean value of $d\,e$ was

$$-0{\cdot}00097840\,;$$

whence we get

$$e = 0{\cdot}002346 05,$$

which coincides to the last figure with the value given in the paper referred to.

On adding, however, the three equations, and substituting this value of e, I found

$$\alpha = -0{\cdot}0019209$$

which differs entirely from the value given for this quantity.

3. Using these values of $d\,e$ and α, I find the following

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elements for this ellipse, and these being the true results of his paper, I shall use as Mr. Pratt's value, distinguishing them and all deductions from his hypothesis by subscribing the letter P to the algebraical representative of the quantity, and calling this hypothesis P.

$$a_{\rm P} = 20882740{\cdot}94 \text{ feet.}$$

$$b_{\rm P} = 20833863{\cdot}65$$

$$e_{\rm P} = 0{\cdot}00234605$$

4. Similarly I shall call the dimensions given by Colonel Everest in his work of 1830 (and which are still in use), hypothesis E, and subscribe that letter in the same way to its results.

$$* \; a_{\rm E} = 20922931{\cdot}80 \text{ feet.}$$

$$b_{\rm E} = 20853374{\cdot}58$$

$$e_{\rm E} = 0{\cdot}003324449$$

5. I first compared these values with Col. Lambton's arc Damergida to Punæ by means of the formula given in Mr. Airy's treatise "On the Figure of the Earth," and also that in Mr. Pratt's paper.

By the former the length

$$l = b\left\{1 + \frac{e}{2} - \frac{3}{2}e \, . \cos(\lambda' + \lambda'') \, . \, \frac{\sin(\lambda' - \lambda'')}{\lambda' - \lambda''}\right\}(\lambda' - \lambda'') \sin 1''$$

and by the latter

$$l = a\left\{1 - \frac{e}{2} - \frac{3}{2}e \, . \cos(\lambda' + \lambda'') \, \frac{\sin(\lambda' - \lambda'')}{\lambda' - \lambda''}\right\}(\lambda' - \lambda'') \sin 1''$$

Now the observed Latitude of Damergida is 18° 3′ 16″·075 for Hyp. E.
And the attraction on hypothesis P ...... + 6·909
∴ The latitude for hypothesis P is ...... 18 3 22·984

Also the observed latitude at Punæ is 8° 9′ 32″·509, which I shall assume without change for both figures. We have then for hypothesis P,

$$\lambda'_{\rm P} + \lambda''_{\rm P} = 26^\circ \; 12' \; 55''{\cdot}493 \qquad \lambda'_{\rm P} - \lambda''_{\rm P} = 9^\circ \; 53' \; 50''{\cdot}475$$

Whence the length is on hypothesis P, by Mr. Airy's formula 3591803 feet.
„ „ „ Mr. Pratt's 3591787 „

And, again, for hypothesis E:

$$\lambda'_{\rm E} + \lambda''_{\rm E} = 26^\circ \; 12' \; 48''{\cdot}584 \qquad \lambda'_{\rm E} - \lambda''_{\rm E} = 9^\circ \; 53' \; 43''{\cdot}566$$

Whence the length is on hypothesis E, by Mr. Airy's formula 3591533 feet.
„ „ „ Mr. Pratt's 3591501 „

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* These lie between the values given by Messrs. Airy and Bessel.

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6. The true length of this arc being 3591779 feet, it would seem that P gives a closer approximation to it. E, however, is nearest to the unaltered observed data; and, the attracting mass lying to the north, it is not in a southerly direction that hypothesis P would fail, if the attractions are erroneous, till both ends of the arc are beyond the limits of sensible attraction. I have only placed these results here to show that the consideration of this arc has not been neglected, as I do not think it can have any weight against that which follows.

7. In the year 1853 a great arc of longitude was completed, extending from Kalianpur to Karachi in Sind, with the following position of its ends:—

	Latitudes.			
	° ′ ″		° ′ ″	
Kalianpur	24 7 11·262	observed.	77 41 44·75	Geodetically computed from Madras as an origin.
Karachi..	24 49 50·155	Geod. comput.	67 4 2·47	

The computations, according to the custom of the department, being made on the figure E, commencing with the place of Kalianpur.

On hypothesis E, therefore, the mean latitude of this arc is—

$$\lambda_E = 24^\circ\ 28'\ 31''$$

and its amplitude

$$\Delta L_E = 10^\circ\ 37'\ 42''\cdot 28$$

8. On hypothesis P, however, the latitude of Kalianpur should be increased by 11″·9, that being the supposed attraction; and as the difference of the latitudes of the extremities will not be materially affected, we shall have

$$\lambda_P = 24^\circ\ 28'\ 43''.$$

9. The value of the length of an arc of longitude is given by Mr. Airy as $= b\,\{1 + e + e \sin{}^2\lambda\}\ \Delta L \sin 1''$, and if we assume the length given in both arcs we shall have

$$\Delta L_P = \Delta L_E \cdot \frac{b_E\,(1 + e_E + e_E\ \sin^2\lambda_E)}{b_P\,(1 + e_P + e_P\ \sin^2\lambda_P)}$$

whence I obtain

$$\Delta L_P = 10^\circ\ 39'\ 2''\cdot 19 = \Delta L_E + 0^\circ\ 1'\ 19''\cdot 81.$$

9. In order, however, to compare the longitude of Karachi with an observed value, we must consider that the amplitude from Madras to the meridian of Kalianpur was derived from a figure nearly approximating to E, though the method was not rigorous. The correction on this account may be assumed as + 22″·5, to suit the hypothesis P.

10. At Karachi I observed on the 22d of May, 1853, an occultation of β_1 *Scorpii*, which I have computed with Mr. Adams'

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value of the lunar parallax and semidiameter. The star's place I took from the Greenwich Results for 1852, and the errors of the Moon's place were interpolated for the mean time from a memorandum kindly furnished by Mr. Airy. Hence I deduced the longitude of my station.

	h	m	s	
By the Immersion..	4	28	3·304	East.
Emersion ..	4	28	5·716	
Mean	4	28	4·510	= 67° 1′ 7″·65 East.

And of the same point reducing to Mr. Taylor's last result for Madras, the longitudes are,

	°	′	″
By hypothesis E ..	67	1	37·64
And by hypothesis P ..	66	59	55·33

E having very decidedly the advantage.

11. The most certain test, however, I have it in my power to apply, is the agreement of the azimuth at Karachi as observed, and that geodetically brought up by computation from Kalianpur.

If the azimuthal angle of B as seen from A be A, and the reverse angle of A seen at B be B, each measured from the north, and we take $B = \pi - (A + \Delta A)$: then, in accordance with Dalby's theorem,

$$\tan \tfrac{1}{2} \Delta A = \frac{\sin \frac{1}{2} (\lambda' + \lambda'')}{\cos \frac{1}{2} (\lambda' - \lambda'')} \tan \tfrac{1}{2} \Delta L$$

Computing, then, the two values of ΔA from the data already given, we shall have between Kalianpur and Karachi

$$\Delta A_E = 4^\circ\ 24'\ 50''\cdot14$$

$$\Delta A_P = 4\ \ 25\ \ 25\cdot16$$

Difference .... 35″·02

which, the computations having been made on hypothesis E, should be the discordance of the azimuths at Karachi, the computed value exceeding the observed one, as the azimuths are measured from south by west; but in a paper on the latitude of Karachi, which I have lately transmitted to the Astronomical Society, I have given the comparison of the observed azimuth at Karachi with that brought up geodetically from Kalianpur, showing that the former is in defect only 0″·312.* No errors of observation, and no possible attraction could account for the enormous amount of

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* The azimuth at Kalianpur cannot be greatly in error, as the discordance in latitude at Karachi I have shown in the same paper to be only 0″·882, and this is the direction we should expect from the neighbouring elevated masses. It must, besides, be considered that Mr. Pratt's attraction would, if it existed, cause the observed azimuth at Kalianpur to be in defect, and thus the error (including this source) would be increased at Karachi.

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35″ in azimuth, and it appears alone sufficient evidence that the hypothesis P is irreconcileable with this arc; unless indeed we were to assume in addition, that the minor axis of the generating ellipse is not the axis of the Earth's revolution, but parallel to it, at such a distance, as to retain the radius of the parallel of latitude.

12. A variation from the general form of the Earth's surface so great as that of hypothesis P is difficult to imagine, for the distance of every point in P from the centre is less than that of the corresponding point in E. The evidence of its existence is so combined with that of deflection of the plumb-line without any real change of form, that it appears impossible to separate entirely these deviations; but the following considerations may, I think, at some future time give a means of deciding on the preferable form.

13. In the first place, the attraction is so enormous, if Mr. Pratt's values hold good, near the mountains, that in longitude it should at Banog Hill Station amount to nearly 20″. If this quantity be called δ L, ν the normal to the minor axis of the spheroid, and λ the latitude of the station; we should expect all the observed right ascensions of the Moon to be affected with a parallax of the form $-\alpha$ P cosec Moon's N.P.D., where $\alpha = \frac{\nu}{a} \cos \lambda \tan \delta L$ (δ L being positive when it tends to increase the east longitude), and after correcting all transits by a term of this form (which, however, would be very small), we should obtain the apparent longitude or inclination of the apparent plane of the meridian to that of Greenwich.

14. The parallaxes, however, of the Moon in right ascension and north polar distance would give means for determining the deflection of the zenith in longitude, as the hour-angle from the hour-circle passing through the production of the Earth's radius would be necessary for their determination. This deflection could consequently be determined from occultations, though of course any determination where the errors bear so large a proportion in all likelihood to the quantity sought, as in this case, must necessarily be uncertain.

15. In the same way a geocentric latitude might be determined from occultations, and compared with an assumed figure, but with greater accuracy. Thus by the Great Arc Series of Triangles the latitude of Banog Hill Station, as brought up geodetically from

	°	′	″
Kalianpur is	[*]30	28	36·96
and that of Kaliana is	[*]29	30	44·78
whence the amplitude is (hypothesis E) ..	0	57	42·18

and on hypothesis E the geocentric latitude of Banog would be 30° 18′ 37″·48.

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[*] These differ from the values given in Colonel Everest's work published in 1847, being uncorrected for the discrepancy in latitude at Kaliana.

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But if hypothesis P be true, the amplitude from Kaliana to

	°	′	″
Banog would be	0	57	45·22
and Mr. Pratt's corrected latitude of Kaliana is ..	29	31	16·75
whence the latitude of Banog should be ..	30	29	1·97
and the geocentric value would be	30	21	58·87

16. The difference between the geocentric latitudes of this point on these hypotheses is then 0° 3′ 21″·39, a large quantity, whose effects would be felt in the parallaxes, for in Admiral Smyth's description of the Bedford Observatory the tenth part of this is spoken of as having a decided effect on the results of occultations.

Possibly by this means we might determine the geocentric latitude within three or four seconds.

17. If an estimate of the attraction at Banog could be made, it would evidently be possible with this and the observed latitude to deduce an angle of the vertex, and an approximate compression of the Earth. The uncertainty of the data, however, and proximity of the attracting masses, would render this result very unsafe, and a preferable proceeding appears to be to determine what constants in the figure of the Earth will deduce the geocentric latitude at the Northern Station from the astronomical latitude of Punæ, which in this investigation we may consider beyond the influence of the mountains.

The compression so obtained would give an astronomical latitude of the Northern Station, which, compared with the observed datum, would show a value of local deflection more trustworthy probably than any resulting from assumed magnitudes, positions and densities of the attracting masses, however carefully these estimates might be made.

I purpose, when my duties in the field allow me, to investigate the necessary formulæ for these determinations.

The instruments at the Lucknow Observatory have, by the annexation of the Kingdom of Oude, become the property of the East India Company. Should it be determined to resume their employment, it may be worth while to remove them to a point on the first Himalayan Range. They might then be employed in determining the longitude close to the axial meridian of India (that of the Great Arc), and in endeavouring to make some estimation of the great local attraction. Situated far above the lower strata of the atmosphere, they would be in a position to examine into the question of "solar refraction" as proposed by Professor C. P. Smyth; and with a telescope of adequate power would be in a more favourable position for micrometrical observations than any other permanently established observatory.

S. G.'s Field Office, Sept. 24, 1856.